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Robust control of aircraft flight in conditions of disturbances

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ABSTRACT

One of the most dangerous parts of the flight is the landing phase, as most accidents occur at this stage. In order to reduce the effect of the low-level wind shear on the longitudinal motion of the aircraft in the glide path landing mode (task) a robust $H-$ control isproposed. Dynamic models of the plane and wind shear are built. H_2 and H_∞ synthesis methods are investigated for the task of aircraft flight control in a vertical plane during landing under conditions of undefined disturbances. Both control methods allow to reduce height deviation significantly. However, suboptimal control H_∞ provides better quality of transition processes both in height and speed than optimal control H_2. The results of simulation of the synthesized system confirm the effectiveness of $H_\infty -$ control for increasing robust stability to uncertainties caused by wind disturbances.

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1. INTRODUCTION

Ensuring flight safety is an urgent problem of modern aviation, especially during flights in difficult meteorological conditions. The most dangerous meteorological phenomenon for aviation flights is low-level wind shear with large gradients of wind components in height and range, which is caused by local disturbances of the atmosphere. Suddenly arising disturbances in the state of the atmosphere are extremely dangerous during an aircraft landing. They led, for example, to two known disasters: at New Orleans International Airport on July 9, 1982 the plane Boeing B-727 crashed during landing and at Dallas International Airport on August 2, 1985 plane Lockheed L-1011 crashed during landing. Due to the great relevance of the problem developers around the world were engaged in the construction of automated flight control systems capable of preventing such catastrophes. Various control algorithms based on different physical principles and mathematical concepts have been proposed, built for different models of the local state of the atmosphere, which somehow solved this problem [1]–[5].

The studies [1], [2], [5] consider the analysis and design of a robust controller. The controller is significant component of an entire automatic landing system developed as part of the aircraft landing task, which was proposed by AIRBUS and ONERA. Techniques of robust synthesis (e.g., structured H_∞ synthesis) acts as an effective basis for accomplishment of these tasks. In research [3] a robust automatic landing controller (SIRAC) based on stable inversion (SI) is proposed. The SI algorithm improves an indicator such as the output tracking accuracy, at the same time the application of the H_∞ synthesis is aimed at increasing the robust stability to uncertainties that arise because of wind disturbances. In research [4] the vertical speed of the aircraft prior to landing is controlled based on a structured $H_\infty -$ control structure,

minimizing the effects of wind shear, ground effects and airspeed changes. A specific multi-model strategy is considered accounting changes in mass and center of gravity location.

In research [6] a pseudo-sliding mode control synthesis procedure is considered and applied to develop control system for a nonlinear NASA Langley generic transport model. The synthesized control system allows minimizing aircraft loss-of-control by maintaining primary pilot input-system response characteristics throughout the flight, taking into account the possibility of actuator damage. In research [7] the problem of robust active fault-tolerant control (FTC) was considered for systems with undefined linear parameter variation (LPV) with simultaneous actuator and sensor failures. In research [8] a parameter independent embedded sliding mode controller with self-adaptation was developed that converges in the system in a finite time, focusing on the uncertain linear parameter variation (LPV) model of the aircraft variant, which has large-scale sweep angle variation and expansion. In research [9] a robust control scheme for commercial aircraft is presented. The control law is supplemented with a prior information about wind gusts. The main contribution of this article is the integration of the wind gust alleviation system using light detection and ranging (LIDAR) with the widely used flight control architecture the so-called C* control law. In research [10] the active vibration control of composite panels with uncertain parameters in a hypersonic flow is studied using the non-probabilistic reliability theory. Using the piezoelectric patches as active control actuators, dynamic equations of panel are determined by the finite element method and Hamilton's principle. The results of the research prove the fact that the control method influenced by reliability, H_∞ performance index, and approach velocity is effective for the vibration suppression of panel throughout an entire interval of uncertain parameters.

The research [11] developed a multi-loop controller for a morphing aircraft to guarantee stability of the wing-shaped transition response. The suggested controller uses a set of inner-loop gains to ensure stability using classical techniques, whereas a gain of self-adaptive H_∞ outer-loop controller is designed to provide a certain level of robust stability and performance of the time-varying dynamics. The paper [12] describes an analysis method, a generalization of the developed parameters of the robust controller for aircraft lateral control using auxiliary damping automatic devices (ADAD). The H_∞ and μ methods served as the basis for performing the synthesis of the proposed controller. The research [13] considers the μ-synthesis procedure for developing a robust autopilot. The software-in-the-loop (SIL) verifications applying blade element theory (BET) confirms that the autopilot is capable to navigate and land the plane in conditions of strong fluctuations in parameters and powerful winds. In studies [14]–[16] Lyapunov functions are used to construct robustly stable control systems. The Lyapunov function is constructed in the form of a vector function, the anti-gradient of which is set by the components of the velocity vector of the system. Some researches [17]–[19] are devoted to the synthesis of robust controllers of aircraft motion parameters using the H_∞ technique. Works [20]–[22] consider the problems of constructing a robust control of the aircraft under the action of uncontrolled disturbances, in which the so-called weight functions are introduced.

$H_\infty -$ control theory is widely used in motion control tasks. The modern period of development of control theory is characterized by the setting and solution of problems, taking into account the inaccuracy of mathematical model of the control object and external disturbances affecting on it. The robust control allows to eliminate indicators such as external disturbances and internal parametric uncertainties. The idea of $H_\infty -$ synthesis is to ensure the stability of a closed-loop system not only for a nominal (without model errors) object, but also for a "disturbed" object (taking into account the model uncertainties and disturbances affecting on the control object) [23]–[25].

This paper investigatesH_2 and H_∞ synthesis techniques for the aircraft flight control problem in the vertical plane during landing in conditions of undefined disturbances. The application of $H_\infty -$ control is effective for increasing robustness to uncertainties caused by wind perturbations. This document is organized as: section 2 describes the principles of stabilization when using H-controls, the synthesis algorithms for $H_2 -$ optimal and $H_\infty -$ suboptimal controls, and builds a mathematical model of the aircraft movement in the vertical plane with regard to wind disturbances, section 3 presents a mathematical model of a vortex ring-shaped wind microburst. The results of H_2 and H_∞ synthesis methods for the problem of aircraft flight control during landing under uncertain perturbations are presented. The efficiency of $H_\infty -$ control is confirmed by the results of simulation of the synthesized system, and section 4 presents the main conclusions of this article.

2. RESEARCH METHOD

2.1. Mathematical model of the longitudinal motion of an airplane

Kinematic and dynamic variables for the equations of motion of the center of mass of the airplane are shown in Figure 1. In Figure 1: X is the drag force; x, y is axes of the coordinate system; Y is the lifting force; O is the center of mass of the aircraft; V is the airspeed of the aircraft; V_e is the aircraft ground speed;

w_x is the horizontal component of wind speed; w_y is the vertical component of wind speed; α is the angle of attack; θ is the angle of inclination of the trajectory in the air coordinate system. The dynamic equations of the aircraft motion in the vertical plane taking into account wind disturbances in projections on the axes of the air coordinate system are set by the system of nonlinear differential (1) [26]–[28]:

$$
\begin{aligned}
&m\dot{V} = Tcos\alpha - X - mgsin\theta - m\left(\dot{w}_x cos\theta + \dot{w}_y sin\theta\right);\\
&mV\dot{\theta} = Psin\alpha + Y - mgcos\theta + m\left(\dot{w}_x sin\theta + \dot{w}_y cos\theta\right);\\
&J_z\dot{\omega}_z = M_z;\\
&\dot{\vartheta} = \omega_z;\\
&\dot{h} = Vsin\theta + W_h(x,h)\\
&\Delta\dot{T} = \frac{1}{\text{Т}_{\text{ДВ}}}\left(-\Delta T + \text{К}_{\text{ДВ}}\Delta\delta_t\right)
\end{aligned} \tag{1}
$$

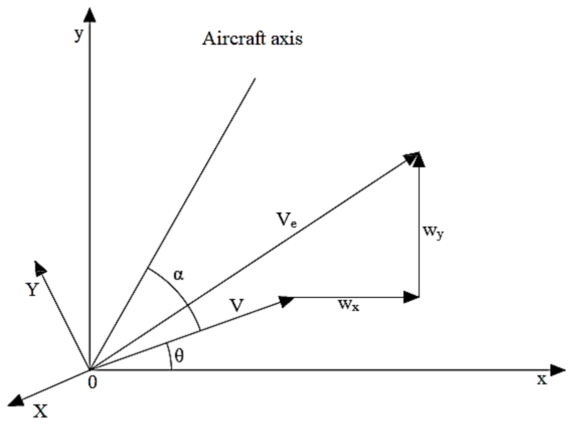

Figure 1. Airplane coordinate system and variables

The control variables are the thrust T and the angle of attack α, which depend, respectively, on the deflection of the engine throttle and the elevator. Here δ_t is the engine throttle deflection from the specified value. As a result of linearization [29], [30] the nonlinear model of the aircraft (1) is reduced to a linear system of ordinary differential equations in increments, which in matrix form has the form (2):

$$\dot{x} = Ax + B_{1n}w + B_{2n}u, \tag{2}$$

where $x = (\Delta V,\ \Delta\theta,\ \Delta w_z,\ \Delta\vartheta,\ \Delta h,\ \Delta T)^T$ = the state vector,
$w = \left(w_y, \dot{w}_x, \dot{w}_y\right)^T$ = the wind disturbance vector,
$u = (\Delta\delta_e, \Delta\delta_t)^T$ = the control vector.

The equation for the measured output y in the state space model in the presence of measurement noise n_y is written as:

$$y = C_y x + I_y n_y,$$

where C_y is the measured output matrix and I_y is the identity matrix of the corresponding dimension. Thus, the mathematical model of the longitudinal motion of the aircraft with considering external wind disturbances in the state space model is described by the system (3):

$$\begin{cases} \dot{x} = Ax + B_{1n}w + B_{2n}u, \\ \quad y = C_y x + I_y n_y. \end{cases} \tag{3}$$

The vector of controlled outputs z_l for a linear model of the longitudinal motion of an aircraft taking into account wind disturbances in the state space model (3) has the form (4):

$$z_l = C_z x. \tag{4}$$

Consider a vector of controlled outputs $\bar{z}$ which is defined as (5):

$$\bar{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} z \\ u \end{bmatrix}. \tag{5}$$

Combining (3), (4) and (5), a system of equations describing the controlled system is obtained:

$$\begin{cases} \dot{x} = Ax + B_{1n}w + B_{2n}u, \\ z_1 = C_z x, \\ z_2 = I_u u, \\ y = C_y x + I_y n_y. \end{cases} \tag{6}$$

The system of equations describing a standard object in the state space model for an extended vector of controlled outputs $\bar{z} = (z^T, u^T)^T$ and an extended vector of external inputs $\overline{w} = (w^T, n_y^T)^T$ has the form

$$\begin{cases} \dot{x} = Ax + B_1\overline{w} + B_2 u, \\ \bar{z} = C_1 x + D_{11}\overline{w} + D_{12}u, \\ y = C_2 x + D_{21}\overline{w} + D_{22}u, \end{cases} \tag{7}$$

where

$$B_1 = [B_{1n} \quad 0], B_2 = B_{2n}, C_1 = \begin{bmatrix} C_z \\ 0 \end{bmatrix}, C_2 = C_y, D_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, D_{12} = \begin{bmatrix} 0 \\ I_u \end{bmatrix}, D_{21} = [0 \quad I_y], D_{22} = 0.$$

2.2 Robust *H* - control

The algorithms for solving the problems of building H_2 – optimal and H_∞– suboptimal controls are considered in this part. Let a finite-dimensional linear controlled and observed object be identified by experimental data in the state space model in the form (8) [23], [24]:

$$\begin{cases} \dot{x}(t) = Ax(t) + \mathrm{B}_1 w(t) + \mathrm{B}_2 u(t) \\ z(t) = \mathrm{C}_1 x(t) + \mathrm{D}_{12} u(t) \\ y(t) = \mathrm{C}_2 x(t) + \mathrm{D}_{21} w(t) \end{cases} \tag{8}$$

where $x(t)$ is the vector of system states; $u(t)$ is the control vector; $w(t)$ is the uncertainty vector, characterizing the inaccuracy of the model; $y(t)$ is the vector of measured outputs; $z(t)$ is the vector of controlled outputs of the system; $A, \mathrm{B}_1, \mathrm{B}_2, \mathrm{C}_1, \mathrm{C}_2, \mathrm{D}_{11}$ and D_{21} is the constant matrices of corresponding dimensions.

The structural diagram shown in Figure 2 represents the synthesized system. Matrices $K(s)$ and $G(s)$ are the transfer matrices of the controller and control object respectively. The matrix $G(s)$ has the structure (9):

$$G(s) = \begin{bmatrix} \mathrm{G}_{11} & \mathrm{G}_{12} \\ \mathrm{G}_{21} & \mathrm{G}_{22} \end{bmatrix} = \begin{bmatrix} A & | & \mathrm{B}_1 & \mathrm{B}_2 \\ - & | & - & - \\ \mathrm{C}_1 & | & 0 & \mathrm{D}_{12} \\ \mathrm{C}_2 & | & \mathrm{D}_{21} & 0 \end{bmatrix} \tag{9}$$

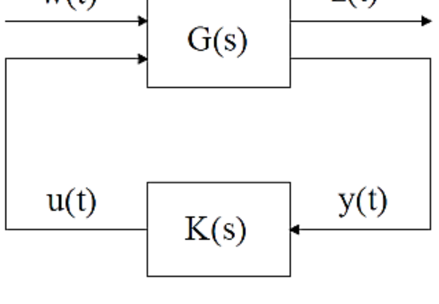

Figure 2. Stabilization principle when using H-control

Robust control is included in the lower subsystem loop $[G_{21}G_{22}]$. Uncontrolled signals passing through the upper subsystem $[G_{11}G_{12}]$ must be effectively suppressed by it. Let the matrix of transfer functions from input $w(t)$ to output $z(t)$ as shown in Figure 2 of the closed-loop system has the form (10):

$$\mathrm{T}_{wz} = A + [\mathrm{B}_1\mathrm{B}_2](I - K\begin{bmatrix}\mathrm{D}_{11} & \mathrm{D}_{12}\\ \mathrm{D}_{21} & \mathrm{D}_{22}\end{bmatrix})^{-1}K\begin{bmatrix}\mathrm{C}_1\\ \mathrm{C}_2\end{bmatrix} \tag{10}$$

The limitation (rationing) T_{wz} is very important. The functional spaces L_2 (the space bounded by the square of the function) and L_∞ (the space of essentially bounded functions) are considered to describe and define the norms T_{wz} [23].

Robust H_2 and $H_\infty -$ controls are sought in the form of feedbacks $u(t) = Ky(t)$ such that the signal norms in the spaces L_2 and L_∞, respectively, equal to $\|T_{wz}\|_2$ and $\|T_{wz}\|_\infty$ are minimal.

$$\|\mathrm{T}_{wz}\|_2^2 = \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} tr\{T_{wz}^T(-j\omega)\cdot \mathrm{T}_{wz}(j\omega)\}d\omega\right)^{1/2} < \infty \tag{11}$$

$$\|\mathrm{T}_{wz}\|_\infty = \sup_{-\infty<\omega<\infty}\bar{\sigma}\ \{\mathrm{T}_{wz}(j\omega)\} < \gamma \tag{12}$$

Where $\|\cdot\|$ is the norm in Hardy functional space; $W(j\omega) = w(p)|_{p=j\omega}$ is the system frequency response; tr is the matrix trace; $sup\bar{\sigma}\{W(j\omega)\}$ is the maximum singular value of the matrix $W(j\omega)$.

The value $\|\mathrm{T}_{wz}\|_2^2$ means the signal energy, and $\|\mathrm{T}_{wz}\|_\infty$ means its intensity in many physical applications. Hence, L_2 is the space of signals of limited energy, and L_∞ is the space of signals of limited intensity. The energy of the error signal under the worst possible perturbation is minimizing while minimizing the norm $\|\mathrm{T}_{wz}\|_\infty$. According to [24], the $H_2 -$ control equations can be written in the form of an optimal observer and an optimal control shaper.

$$\begin{cases}\dot{\hat{x}}(t) = A\hat{x}(t) + \mathrm{B}_2u(t) + \mathrm{L}_2(\mathrm{C}_2\hat{x}(t) - y(t))\\ \qquad u(t) = \mathrm{F}_2\hat{x}(t)\end{cases} \tag{13}$$

Where F_2 is the matrix of the gain coefficients of the optimal $H_2 -$ the control feedback; L_2 is the matrix of the optimal feedback gain coefficients by H_2 observation.

$H_\infty -$ control has the form (14):

$$\begin{cases}\dot{\hat{x}}(t) = A\hat{x}(t) + \mathrm{B}_1\widehat{W}_{worst}(t) + \mathrm{B}_2u(t) + \mathrm{Z}_\infty\mathrm{L}_\infty(\mathrm{C}_2\hat{x}(t) - y(t))\\ \qquad u(t) = \mathrm{F}_\infty\hat{x}(t)\\ \qquad \widehat{W}_{worst}(t) = \gamma^{-2}B_1^TX_\infty\hat{x}(t)\end{cases} \tag{14}$$

where F_∞ is the matrix of the gain coefficients of the optimal $H_\infty -$ the control feedback; $\mathrm{Z}_\infty\mathrm{L}_\infty$ is the matrix of the optimal feedback gain coefficients by H_∞ observation. It can be seen from these equations that unlike the $H_2 -$ observer, the $H_\infty -$ observer has anobserver-based compensator structure (because of the $\mathrm{B}_1\widehat{W}_{worst}(t)$ component). The main differences in the control structure are the appearance of a new structural component $\mathrm{B}_1\widehat{W}_{worst}(t)$ in the $H_\infty -$ case and the replacement of L_2 by $\mathrm{Z}_\infty\mathrm{L}_\infty$.

H_2-optimal control can be constructed in a finite number of operations. However, it is necessary to make a reservation that the real program uses iterative procedures to solve the algebraic Riccati equation, which makes this statement true if the procedure for solving the Riccati equation considered as a separate operation. The H_2-optimal control synthesis algorithm has a linear structure as shown in Figure 3. Unlike the $H_2 -$ case $H_\infty -$ suboptimal control (like $H_\infty -$ norm) cannot be defined by a finite number of operations and requires an iterative procedure.

The synthesis algorithm of $H_\infty -$ control, shown in Figure 4 (in appendix), has a branched structure, this is explained by the need to check the condition $\rho(\mathrm{X}_\infty\mathrm{Y}_\infty) < \gamma^2$ and find γ with the required degree of accuracy ε. If the condition is not met, it is necessary to enter a new value of γ larger than the previous one; if the accuracy condition $|\rho - \rho_0| < \varepsilon$ is not met, where ρ_0 is the spectral radius at the previous value of γ and ρ is for current value, it is necessary to enter a new γ smaller than the previous one. This algorithm implements the input of γ at each step manually. Construction of the control is carried out already at the selected value of γ and the corresponding matrices and X_∞ and Y_∞. It is seen that the synthesis of $H_\infty -$ control is much more labor-intensive than the synthesis of $H_2 -$ control also because it is necessary to solve two Riccati equations in each cycle of choosing γ, while for the $H_2 -$ case these equations are solved once.

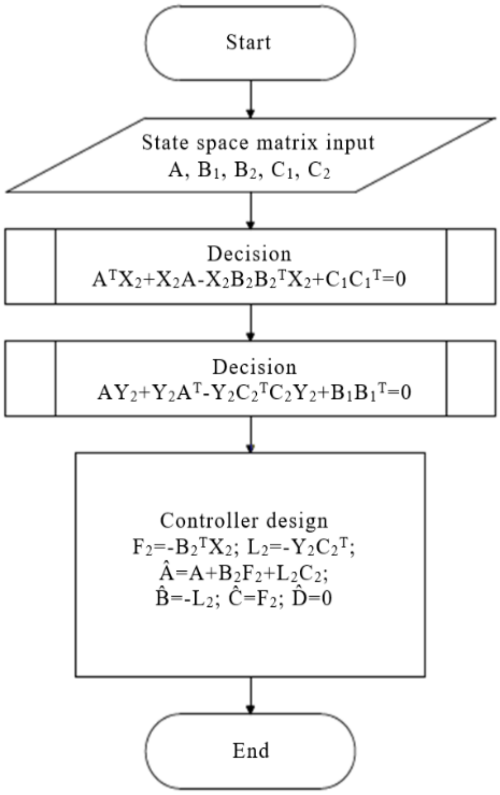

Figure 3. Flowchart of the H_2 − optimal control synthesis algorithm

3. RESULTS AND ANALYSIS

3.1. Mathematical model of a wind microburst in the form of a vortex ring

Wind shear is a change in the magnitude and direction of the wind speed during the transition from one point in space to another, related to the distance between the points under consideration. The aircraft is strongly disturbed during wind shear, therefore, in the take-off and landing modes (due to the low flight altitude), a dangerous situation can quickly arise without vigorous and timely intervention of the pilot in the aircraft control. Piloting an aircraft in wind shear conditions is challenging due to the need to simultaneously control the elevator (balancing the aircraft) and the engine thrust (recovering speed). All of this has led to the fact that wind shear at low altitude has become a dangerous phenomenon. Mathematical models of wind microbursts of varying complexity have been developed for research and development of automatic or manual control systems.

The generated wind profiles of this model allow simulating atmospheric conditions corresponding to some real-life situations, in particular, at Dallas (1985) and New Orleans (1982) airports. According to this model [3], [30], the wind microburst area is formed by the flow around a vortex ring located above a flat surface. The geometric relations are shown in Figure 5. The mathematical model of a vortex ring-shaped wind microburst is described in (15):

$$\psi = \frac{\Gamma}{2\pi}(r_1 + r_2)[F(\lambda) - F(\lambda)], \tag{15}$$

where Γ is the circulation, R is the radius of the filament of the vortex ring, $F(\lambda)$, $F(\lambda)$ are the complete elliptic integrals of the first and second kind, r_1,r_2 are the largest and smallest distances from the current point (x, z, h) to the filament of the vortex ring, R_c is the effective radius of the vortex ring core and a dimensionless variable.

$$\lambda = \frac{r_2 - r_1}{r_2 + r_1}.$$

The final formulas for wind speeds at the points in space with coordinates (x, h) are as:

$$w_x = \frac{1{,}128\rho\Gamma}{2\pi}\left\{\frac{R}{((x-X)^2+h_\rho^2+R^2)^{1/2}}\left(\frac{h_\rho}{r_{2\rho}}-\frac{h_\rho}{r_{1\rho}}\right)-\frac{R}{((x-X)^2+h_m^2+R^2)^{1/2}}\left(\frac{h_m}{r_{2m}}-\frac{h_m}{r_{1m}}\right)\right\},$$

$$w_y = \frac{1{,}576\rho\Gamma}{2\pi}\left\{\frac{R}{(\frac{1}{4}(x-X)^2+h_\rho^2+R^2)^{3/4}}\left(\frac{x_1}{r_{1\rho}^{3/4}}-\frac{x_2}{r_{2\rho}^{3/4}}\right)-\frac{R}{(\frac{1}{4}(x-X)^2+h_m^2+R^2)^{3/4}}\left(\frac{x_1}{r_{1\rho}^{3/4}}-\frac{x_2}{r_{2\rho}^{3/4}}\right)\right\}$$

where

$$x_1 = x - X - R, h_\rho = h - H, r_{1\rho} = x_1^2 + h_\rho^2, r_{2\rho} = x_2^2 + h_\rho^2,$$

$$x_2 = x - X + R, h_m = h + H, r_{1m} = x_1^2 + h_m^2, r_{2m} = x_2^2 + h_m^2, r_0 = \min(r_{1\rho}, r_{2\rho}), \rho = 1 - e^{\frac{r_0}{R_c}}.$$

The graphs of the vertical w_y and horizontal w_x components of the wind profile relative to the vortex center in the wind microburst zone at a flight altitude of 400 m with parameters (Γ=45000 m^2/s, R=350 m, R_c=35 m, H=700 m) are shown in Figure 6.

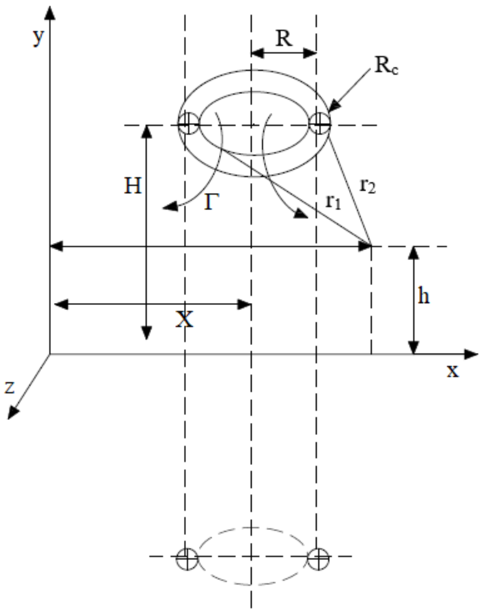

Figure 5. Microburst area

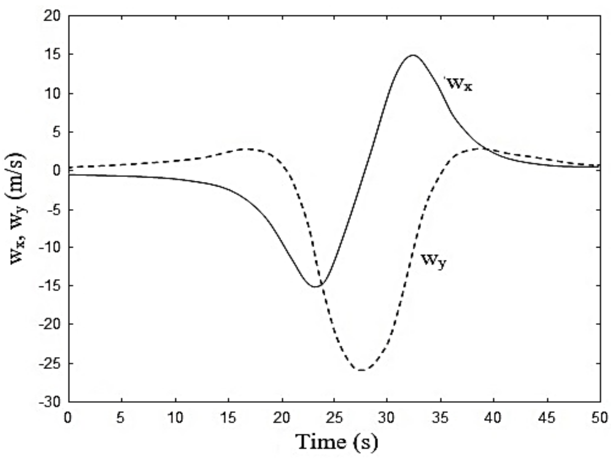

Figure 6. Graphs of vertical w_y and horizontal w_x components of the wind gust profile

3.2 Simulation results

This part considers the specific trajectory of the aircraft glide path [30]–[32]. This trajectory in coordinates of altitude and range is a straight line with a given trajectory inclination angle Θ_{gl} (Θ_{gl}= -2.7 degrees). The task of the control system is to provide a constant air speed V_0 = 71.375 m/s and a given height h = 400 m when moving along the glide path under the action of wind disturbances, the model of which is described above.

The results of a comparison of the quality of transient processes of closed-loop systems with the above described $H_2 -$ and $H_\infty -$ controls constructed using the proposed methodology are presented further. During the simulation, the same signal was applied to the input of each closed-loop system, simulating the wind disturbance $\overline{w}$acting on the aircraft when it moves in the zone of the wind microburst. Figure 7 shows the graphs of the deviation of the airspeed V from the nominal value for two controls. The maximum speed deviation when using $H_2 -$ control is about 3.49 m/s, and when using $H_\infty -$ suboptimal control is about 1.375 m/s. $H_\infty -$ suboptimal control provides better quality of transient processes than $H_2 -$ control according to the airspeed deviation.

Figure 8 shows the graphs of the deviation of the altitude h from the nominal value for two controls. $H_\infty -$ suboptimal control also provides better quality of transient processes than $H_2 -$ control by deviation of altitude. Then the maximum height deviation when using $H_2 -$ control is about 18.75 m, and when using $H_\infty -$ suboptimal control is about 7.7 m, that is, almost 2.5 times less. This characteristic is very important because a sharp loss of altitude in the microburst zone is the main cause of accidents during aircraft landing.

Thus, it could be concluded that the optimal systems synthesized by the quadratic quality criterion are sensitive to the model parameters of the real object and the characteristics of the input influences, i.e are not robust [32], [33]. The optimal control has been synthesized by taking into account the need to provide a compromise between the minimum possible deviation of the controlled outputs (airspeed and altitude) from the nominal values and the power limitations of the control units (engines and elevators). For this purpose, a value characterizing the control was introduced into the optimality criterion. $H_\infty -$ control solves the problem of minimum sensitivity of the closed-loop system for the worst-case external perturbation. The energy of the disturbance passing to the output is determined by the $H_\infty -$ norm of the matrix transfer function of the closed-loop system from the external disturbance to the controlled output. The idea of robust control synthesis is to provide with one control the stability of a closed-loop system not only for a nominal (without model errors) object, but also for a "perturbed" object (taking into account model uncertainties and perturbations acting on the control object).

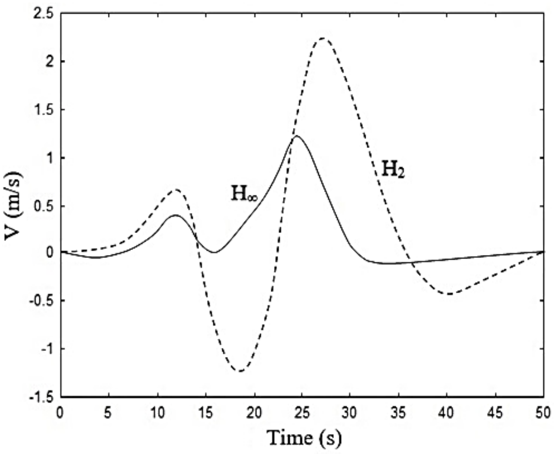

Figure 7. Velocity deviation V when using $H_2 -$ and $H_\infty -$ control

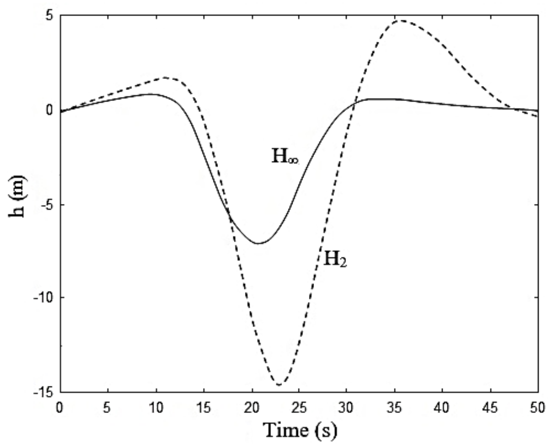

Figure 8. Altitude deviation h when using $H_2 -$ and $H_\infty -$ control methods

4. CONCLUSION

This paper examines H_2 and H_∞ synthesis methods to reduce the influence of low-altitude wind shear on the longitudinal motion of the aircraft in the glide path landing mode. A mathematical model of aircraft movement in the vertical plane with considering wind disturbances, and a mathematical model of a wind microburst in the form of a vortex ring were obtained. Both controls allow for a significant reduction in altitude deviation. However, the $H_\infty -$ suboptimal control method provides better quality of transient

processes both in altitude and speed than the $H_2 -$ optimal control. Consequently, the $H_\infty -$ control provides significantly better suppression of external wind disturbance. Further research by the authors will focus on the use of mixed $H_2/H_\infty -$ control, which allows to obtain the average quality of regulation. Despite a much more complicated algorithm for its calculation, difficulties in selecting weighting coefficients and the level of γ, it will probably provide a wide range of transients, each of which in certain casesmay be more useful than the results of optimization by a single criterion. Eventually the implementation of mixed $H_2/H_\infty -$ control may create a more comfortable on-board environment than the $H_\infty -$ suboptimal system.

APPENDIX

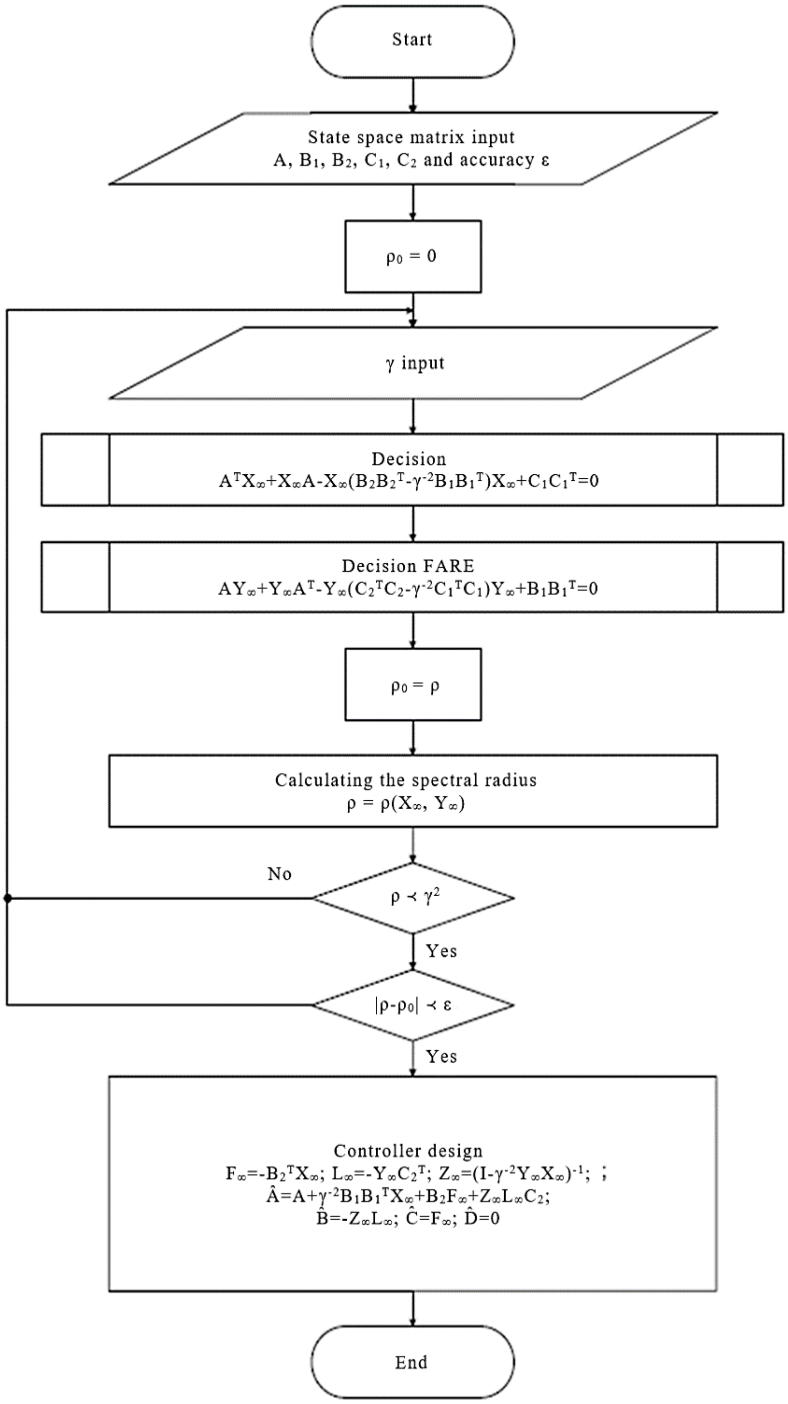

Figure 4. Flowchart of the $H_\infty -$ suboptimal control synthesis algorithm

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